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Dispersion and Leakage Characteristics of Coplanar Waveguides

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Abstract—**The spectral-domain approach is utilized to discuss the dispersion and leakage phenomenon in a coplanar waveguide structure caused by the substrate surface wave. In this study, the effective dielectric constant and the attenuation constant due to surface wave leakage are presented and discussed in detail.**

I. Introduction

Recently the coplanar waveguide structure receives increased attention due to its potential applications in millimeter wave spectrum. With all three conductors on the same side of the substrate, the coplanar waveguide is easy in adaptation to active and passive components in shunt and series configurations and hence becomes a useful component of millimeter-wave integrated circuits.

The coplanar waveguide structure was proposed by Wen [1] as a transmission medium in microwave circuits. Its dispersion characteristics were studied, using the full-wave analyses such as spectral-domain approach [2] and hybrid approach [3].

The possibility of leakage in coplanar waveguide structure through substrate surface-wave modes was discussed and estimated by a simplified theory based on reciprocity [4], [5]. Leakage to substrate surface-wave modes was also observed in other structures such as the coplanar stripline [6], the slot line [7], [8], the microstrip on an anisotropic substrate [9], and the conductor-backed coplanar waveguide [5]. Recent leakage study on coplanar waveguides of finite and infinite widths revealed several interesting behaviors such as sharp and deep minima and narrow sharp peaks [10]. Since power leakage through surface waves may produce undesired cross talk and package effects, there is a need of detail leakage analysis for the coplanar waveguide structure.

In this study, the spectral-domain analysis will be utilized to discuss the leakage phenomenon in an open coplanar waveguide structure caused by the substrate surface wave. The dispersion and leakage characteristics of the coplanar waveguide will then be discussed in detail, which include typical numerical results such as the effective dielectric constant and the attenuation constant due to surface wave leakage.

II. Spectral-Domain Analysis

Consider the coplanar waveguide structure (insert of Fig. 1) with strip width w, slot width, s, and a substrate of thickness h and relative dielectric constant ϵ_r. It is assumed that all field quantities are of the form $\exp[j(\omega t - k_z z)]$. To conduct the spectral-domain analysis, the Fourier transformation pair is introduced as

$$\tilde{A}(k_x) = \int_{-\infty}^{\infty} A(x) e^{-jk_x x}\, dx$$

$$A(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{A}(k_x) e^{jk_x x}\, dk_x. \tag{1}$$

Then a relation which relates electric currents $(\tilde{J}_z, \tilde{J}_x)$ to electric fields $(\tilde{E}_z, \tilde{E}_x)$ in the spectral domain can be established [11]

$$\begin{pmatrix}\tilde{J}_z\\ \tilde{J}_x\end{pmatrix} = \begin{pmatrix}\tilde{G}_{zz} & \tilde{G}_{zx}\\ \tilde{G}_{xz} & \tilde{G}_{xx}\end{pmatrix}\begin{pmatrix}\tilde{E}_z\\ \tilde{E}_x\end{pmatrix}. \tag{2}$$

Here $\tilde{G}_{zz}$, $\tilde{G}_{zx}$, $\tilde{G}_{xz}$, and $\tilde{G}_{xx}$ are the transformed Green's functions whose poles may be identified with the characteristic surface wave modes of the dielectric slab with back-side metallization.

In this analysis, the tangential electric fields on the slot are expanded as

$$E_z(x) = \sum_{\nu} C_z^{\nu}\, \Phi_z^{\nu}(x)$$

$$E_x(x) = \sum_{\nu} C_x^{\nu}\, \Phi_x^{\nu}(x), \tag{3}$$

where C_z^{ν} and C_x^{ν} are unknown coefficients to be determined and $\Phi_z^{\nu}(x)$ and $\Phi_x^{\nu}(x)$ are known basis functions as suggested by [12]. By applying the Galerkin's procedure in the spectral domain, the following matrix equation can be derived

$$[Z_{ij}^{\nu\mu}][C] = 0 \tag{4}$$

where

$$[C] = [C_z^{\nu} C_x^{\nu}]^t$$

$$Z_{ij}^{\nu\mu} = \int_{-\infty}^{\infty} \tilde{\Phi}_i^{\nu}\, \tilde{G}_{ij}^{\nu\mu}\, \tilde{\Phi}_j^{\mu}\, dk_x, \qquad i, j \in \{x, z\}. \tag{5}$$

The propagation constant k_z is then obtained by requiring the determinant of the Z-matrix be zero, and the effective dielectric constant $\epsilon_{\text{eff}} = (k_z/k_0)^2$ can be achieved.

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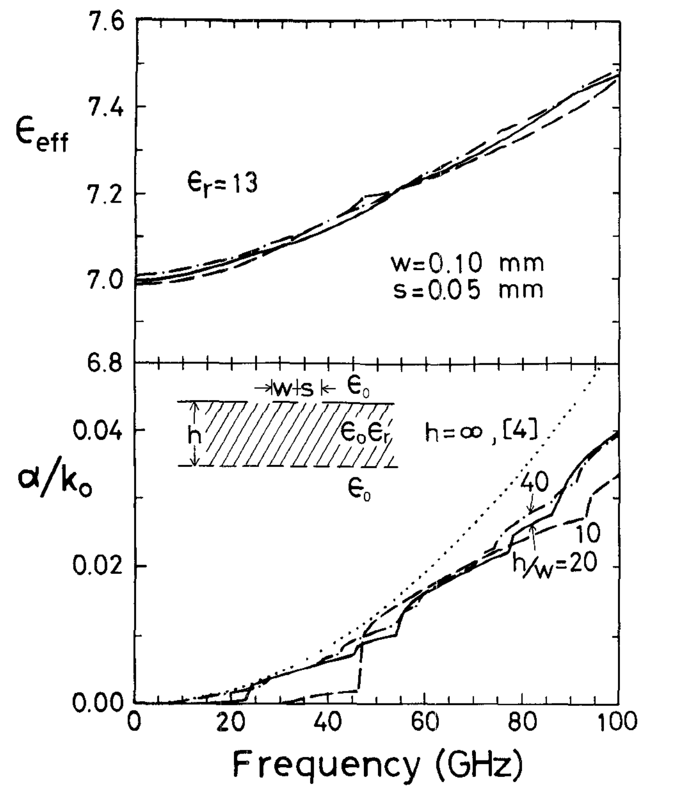

Fig. 1. Comparison fo normalized attenuation constants by reciprocity [4] and spectral-domain analysis.

III. Leakage by Surface Waves

The effective dielectric constant of the coplanar-waveguide mode increases as frequency does, but the effective dielectric constant of the substrate surface wave increases faster than that of the coplanar-waveguide mode. Above some critical frequency the effective dielectric constant of coplanar-waveguide mode may be less than that of the substrate surface wave which will then be excited. This substrate surface wave carries a power in the direction other than z-axis, and the power of coplanar-waveguide mode guided along z-axis will decay. Therefore, one must regard the propagation constant of the guided mode as complex and call the mode as the leaky guided-mode of coplanar waveguide.

In order to solve this leakage problem, one should consider a current distribution of the form

$$\vec{J}_s(x, z) = \vec{J}_s(x) e^{-j(\beta - j\alpha)z}. \quad (6)$$

It is also assumed that this decaying current gives rise to the same decaying behavior in the fields. Hence, the propagation constant $k_z = \beta - j\alpha$ is complex, and α corresponds to the attenuation due to leakage from surface wave.

In the leakage problem the surface wave will excite an exponential-growth field of the form $e^{p''|x|}$; for instance $A \to e^{-jp_+x}$ as $x \to \infty$ and $A \to e^{jp_-x}$ as $x \to -\infty$ where $p_\pm = p'_\pm + jp''_\pm$, $p'_\pm$ and $p''_\pm > 0$. Following the method presented in [13], [14], the inverse Fourier transform should be modified as

$$A(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{A}(k_x) e^{jk_x x}\, dk_x + \frac{2\pi j}{2\pi}\{\text{Res}\,[\tilde{A}(k_x), -p_+] - \text{Res}\,[\tilde{A}(k_x), p_-]\}. \quad (7)$$

Mathematically, the complex numbers $-p_+$ and p_- are poles of the complex function $\tilde{A}(k)$, and Res $[\tilde{A}(k_x), -p_+]$ means the residue of $\tilde{A}(k_x)$ at $-p_+$. In the spectral-domain analysis, since the basis functions exist only on the slot, the transforms of these basis functions are always analytic over the entire complex spectral plane. Thus the integration contours are determined by the poles ($-p_+$ and p_-) of Green's functions only. The spectral integrals in (5) should be modified as

$$Z_{ij}^{\nu\mu} = \int_{-\infty}^{\infty} \tilde{\Phi}_i^{\nu}\, \tilde{G}_{ij}^{\nu\mu} \tilde{\Phi}_j^{\mu}\, dk_x + 2\pi j\{\tilde{\Phi}_i^{\nu}(p_+) \cdot \text{Res}\,[\tilde{G}_{ij}^{\nu\mu}(k_x), -p_+]\, \tilde{\Phi}_j^{\mu}(-p_+) - \tilde{\Phi}_i^{\nu}(-p_-)\, \text{Res}\,[\tilde{G}_{ij}^{\nu\mu}(k_x), p_-]\, \tilde{\Phi}_j^{\mu}(p_-)\}. \quad (8)$$

Finally by solving the complex propagation constant $k_z = \beta - j\alpha$ from the equation det $[Z] = 0$, one may obtain the effective dielectric constant $\epsilon_{\text{eff}} = (\beta/k_0)^2$ and the attenuation constant α due to surface wave leakage.

IV. Numerical Results

A computer program based on the previous theory is developed to analyze the coplanar waveguide structure. For a study of leaky guided-mode, more longitudinal and transverse expansion functions are needed for a convergent result. However, when the substrate wave is not excited, two longitudinal and two transverse expansion functions are adequate. Presented in this study are just the numerical results for the fundamental coplanar-waveguide mode. Although several surface wave modes are excited in the results in Fig. 1, only one surface wave mode (TM_0) is excited in those of Fig. 2 to Fig. 5.

Fig. 1 compares the normalized attenuation constant α/k_0 with that calculated from reciprocity [4]. Note that results of [4] are under the assumption of infinite substrate thickness and our results are for finite thickness. Thus, ours will approach those of [4] when the substrate is thick enough. One interesting observation in Fig. 1 is the slope of the α/k_0 curve is discontinuous whenever a surface wave is excited. Besides, the effect on both ϵ_{eff} and α/k_0 curves due to the excited TE surface wave is larger than that due to the TM surface wave.

Fig. 2 shows the effective dielectric constant and normalized attenuation constant versus frequency with substrate dielectric constant ϵ_r as parameters. In the normalized attenuation curve, a narrow small sharp peak and associated narrow sharp minimum are observed immediately after the onset of leakage, as reported in [10]. It is also found that the α/k_0 curve approaches another maximum (a broad and bigger peak) as frequency goes higher and then decays monotonously. This implies that the coupling between coplanar-waveguide mode and surface wave reaches a maximum at some particular frequency.

Shown in Fig. 3 is the effect of increasing substrate thickness on the effective dielectric constant and normalized attenuation constant. Because the characteristics of the surface waves are mostly decided by the substrate thickness h, the magnitude of the normalized attenuation constant due to the excited surface wave is strongly related to h. It is realized that the thicker the substrate is, the smaller frequency the leaky wave begins, and the larger the maximal attenuation is. Note that two peaks, a narrow sharp peak followed by a broad peak, are also observed in Fig. 3. The sharp peak increases and the broad peak decreases as the substrate thickness decreases, making the sharp peak greater than the broad peak as in the case of $h = 0.635$ mm.

While increasing the strip width has little influence on the effective dielectric constant, as is shown in Fig. 4, it does decrease the

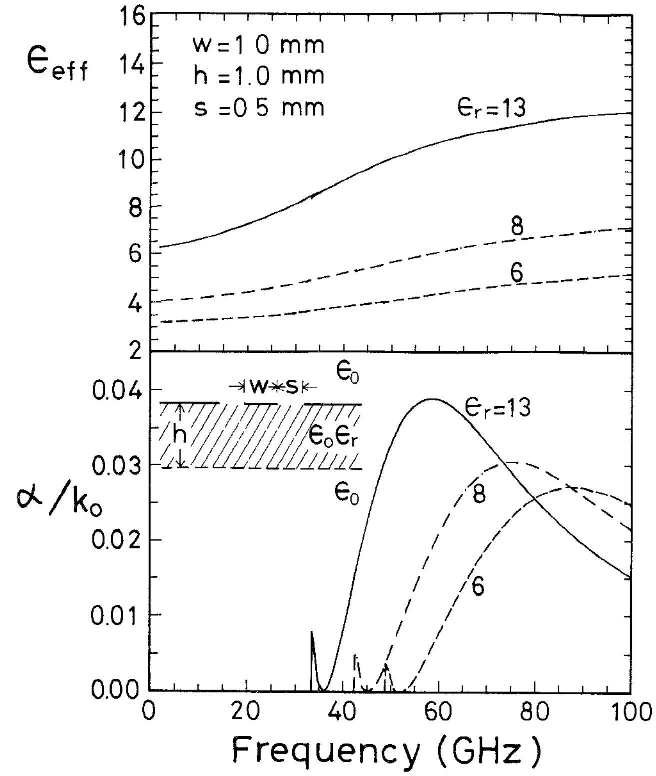

Fig. 2. Effective dielectric constant and α/k_0 versus frequency with substrate dielectric constant ϵ_r as parameters.

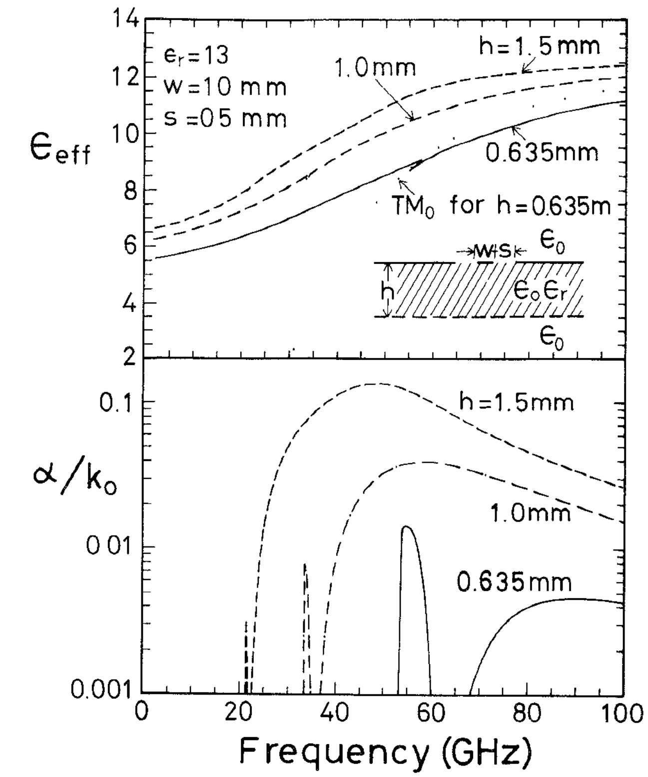

Fig. 3. Effective dielectric constant and α/k_0 versus frequency with substrate thickness h as parameters.

attenuation constant significantly. Thus, one can adjust the strip width for a smaller α/k_0 without changing the effective dielectric constant too much.

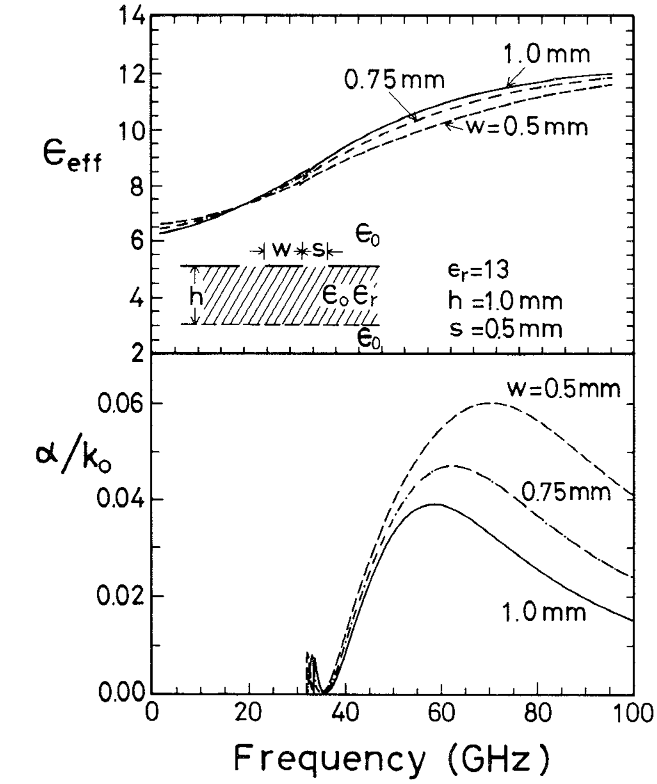

Fig. 4. Effective dielectric constant and α/k_0 versus frequency with strip width w as parameters.

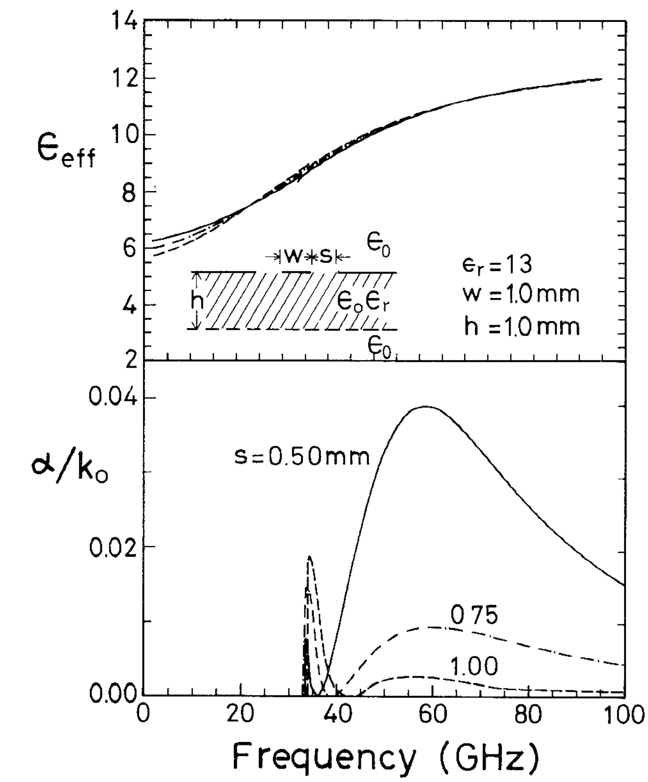

Fig. 5. Effective dielectric constant and α/k_0 versus frequency with slot width s as parameters.

Fig. 5 shows the effective dielectric constant and α/k_0 versus frequency with slot width s as parameters. Note that the sharp peak in the α/k_0 curve increases and the broad peak decreases as the slot width s increases. The sharp peak may again be greater than the broad peak as indicated by the case of $s = 1.00$ mm. Again the effective dielectric constant is not strongly dependent on the slot width.

One conclusion from Fig. 3 to Fig. 5 is that the thickness of the substrate is the most important parameter in affecting the leakage attenuation. The effect of changing slot width is somewhat higher than that of the changing strip width, but both are minor in comparsion with that of substrate thickness.

V. Conclusions

The dispersion and leakage characteristics such as the effective dielectric constant and normalized attenuation constant of coplanar waveguide have been treated using the spectral-domain approach together with the complex residue technique. To handle the complex propagation constant of the coplanar waveguide, the Fourier transform and the Parseval's theorem in complex plane are properly extended. A number of numerical results, such as the effective dielectric constant and normalized attenuation constant, have been presented to illustrate the characteristics of coplanar waveguide. Sharp peaks just after leakage together with broad peaks are two interesting phenomena observed in the attenuation characteristics. The physical mechanism of these peaks and the detail of transition in ϵ_{eff} and α/k_0 curves are still unclear and are worthy of further study.

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The Scattering Matrix Formulation for Overmoded Coaxial Cavities

W. Lawson and P. E. Latham

Abstract—**The scattering matrix formulation for complex right-circular cavities is extended to coaxial circuits with variable inner radii. The modified eigenvectors, which include the TEM wave, and the modified boundary conditions are presented. The properties of several configurations are examined and transmission measurements are shown to be in good agreement with theory.**

I. Introduction

The operation of coaxial transmission lines in frequency bands where only the TEM mode can propagate is straightforward and widespread. However, some potential applications, including high power microwave tubes, require highly overmoded coaxial circuits. Gyrotrons, for example, have utilized coaxial circuits operating in the $TE_{5,2}$ mode [1], and more recently in the $TE_{20,13}$ mode [2]. The principle advantage of coaxial cavities over right circular cavities is the enhancement in stable operation due to the decrease in mode density.

This work describes the calculation of resonant frequencies (real part and diffractive quality factor) for overmoded coaxial cavities. It is an extension of previous results which utilize the scattering matrix approach [3]–[7] to analyze right circular cavities [8]. The addition of the inner conductor results in four complications. First, coaxial systems can support TEM waves. Second, the radial dependence of the fields involve bessel functions of the second kind. A consequence of the second fact is that the perpendicular wave number becomes a complicated function of the inner and outer radii. Finally, variations in the radius of the inner conductor result in additional boundary conditions.

In Section II, the analysis is described briefly and the coefficients of the mode coupling matrix are presented. Typical numerical results, along with some experimental data, are described in Section III. The final section summarizes the results of this work.

II. Theory

Because the scattering matrix approach is described in detail elsewhere [8], only a brief outline is given here. In this approach, a cavity is divided into regions with uniform cross section and the

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